The Influence of Thermal Effect Distributed Vertically over the Large Scale Atmospheric Disturbances

The Influence of Heat Sources and Sinks Distributed Vertically over the Large Scale Atmospheric Disturbances

By Gaishi Onishi
Geophysical Institute, Faculty of Science, Tôhoku University

(Received 19 October 1954)

Abstract

In order to study the thermal effect on the planetary waves, the model considered here is such that, heat sources, represented by the sensible heat, is distributed in the lower atmosphere, and heat sinks, due to the radiative cooling, in the upper layer, and that the motion is frictionless. By numerical computation it is concluded that the thermal effect is so small that it can be neglected in small time intervals. But in view of the maintenance of the long stable planetary waves, the thermal effect seems to be important in amplifying the waves in opposition to the frictional decay effects.

The effect of the stratosphere is also studied. It seems that the single layer approximations with the respective simplifications of the finite atmosphere bounded by two rigid surfaces and of the infinite atmosphere, are not satisfactory for unstable cyclonic waves.

1 Introduction

Many works have been carried out to investigate the large scale disturbances in two or three-dimensional baroclinic atmosphere. Owing to the complexity of the mathematical treatment, it is accustomed to consider a simplified model atmosphere with omission of those details which will not make a major contribution. In general, two alternative assumptions have been made in solving the single layer problem. One is that the whole atmosphere is assumed to be of a same structure neglecting the difference of the troposphre and the stratosphere. The errors which should result from this assumption on the wave motion have been believed to be negligible, because the exponential decrease of density with height should reduce the zonal momentum to a negligible value in the upper layer and should nullify the influence of the static stability in the upper layer. Another is that the tropopause is assumed to be a rigid boundary which is equivalent to a stratosphere of infinite static stability. It is of interest which alternative is more appropriate.

The second simplification is the adiabatic hypothesis, which would be acceptable, if the effects of radiation, turbulent heat transfer and condensation could be neglected. The first two effects may be regareded as of secondary importance in the free atmosphere, whereas the neglect of condensation will produce appreciable errors. Polytropic change hypothesis will diminish this difficulty to some extent, but it will not be considered sufficient.

Lately, J. SMAGORINSKY (1953) investigated the dynamical influence of heat sources and sinks, distributed on a longitudinal extent, on large-scale disturbances such as monsoons, considering the surface friction and the existence of the stratosphere. In the present paper, in order to examine the effect of condensation, an attempt was made to study the influence of heat sources and sinks, distributed not only horizontally but also vertically.

2 The Distribution of Heat Sources and Sinks

The solar radiant energy is an only source of the atmospheric energy and is transformed to the thermal and kinetic energy of the atmosphere. About two third of the incident solar energy is available for heating, directly and indirectly, the earth and the atmosphere. On the other hand the outgoing long-wave radiation to space is an only sink of the atmospheric energy. H. G. HOUGTON (1954) pursued the annual heat balance of the northern hemisphere. By his results, the heat sources are situated in tropical and subtropical regions and sinks in high latitude regions, and at about 40°N the total solar radiation absorbed takes the same value as the outgoing long-wave radiation at the tropopause. It, therefore, is plausible to assume that the heat balance in the vertical column is being kept, on the averge, around the hemisphere at 45°N.

From the point of view of average radiative processes alone, the distribution of the sources and sinks in the atmosphere may be considered to be given. Many investigators (P. RAETHJEN 1950, J. LONDON 1952, 1953, HOUGTON 1954, and etc,) concluded that the surface is a source everywhere except in the winter polar regions and the adjacent continental areas, whereas the troposphere is everywhere a sink, especially at the level of the uppermost cloud surface, (see fig. 1). It is a remarkable result that the sink of heat does not, in the average, occur in the lower stratosphere. So we will assume that the distribution of sources and sinks is limitted only in the

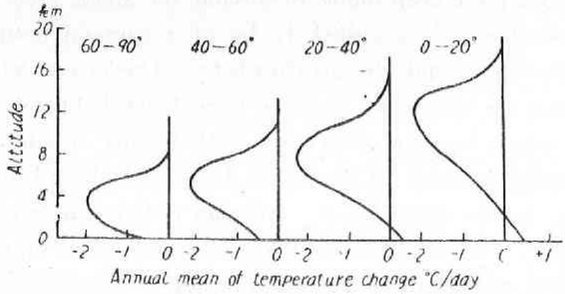

Fig. 1. Radiative temperature changes for various zone of latitutde. (after RAETHJEN)

troposphere. In addition to sources and sinks due to radiation, it is necessary to consider the latent heat of condensation and the eddy flux of heat. The latent heat of condensation can be estimated from the mean precipitation. On account of the scarcity of water vapor in the upper troposphere, the latent heat will play only a minor role in the atmosphere higher than 5 km. A minute study on the eddy flux of heat has not been made. The flux is directed upwards but its magnitude is still in debate.

The distibution of sources and sinks is assumed as follows :

$$\frac{1}{c_p T}\frac{dq}{dt} = Q_1 + Q_2\,, \tag{1}$$

where T is the temperature, c_p is the specific heat of air at constant pressure, dq/dt is the rate of change of heat per unit mass, and Q_1 is the perturbation heat sources and sinks considered by SMAGORINSKY, represented by

$$Q_1 = N e^{-z/h} (\sin \pi\, z/z_T) \sin kx \sin \mu y\,,$$

where the rectangular system of coordinate x, y and z is introduced with x increasing eastward, y north-ward and z vertically upward, and z_T is the height of the tropopause. It is noticed that the mean value of Q_1 at any altitude, averaged over vertical direction, becomes zero, so that it seems to be inadequate to represent the characteristics of the vertical distribution of sources and sinks by Q_1 alone. Hence Q_2 is introduced in the present investigation which represents the vertical distribution of sources and sinks, sources in the lower half and sinks in the upper half of the troposphere, independently of longitude, as will be shown more precisely later.

We will try to obtain the solution for the perturbation Q_2 alone, in the following research. The complete solution corresponding to equation (1) is, then, given by the sum of the solution and that obtained by SMAGORINSKY.

3 Theoretical Equations

The vorticity equation may be written similarly to H. L. KUO's (1949) derivation. Neglecting the curvature of the earth, the Eulerian equations of motion become

$$\left\{\begin{aligned}
&\frac{du}{dt} = -s\frac{\partial p}{\partial x} + fv\,,\\
&\frac{dv}{dt} = -s\frac{\partial p}{\partial y} - fu\,,\\
&\frac{\partial p}{\partial z} = -\frac{g}{s}\\
&\frac{\partial}{\partial t}\left(\frac{1}{s}\right) + \frac{\partial}{\partial x}\left(\frac{u}{s}\right) + \frac{\partial}{\partial y}\left(\frac{v}{s}\right) + \frac{\partial}{\partial z}\left(\frac{w}{s}\right) = 0\,.
\end{aligned}\right. \tag{2}$$

where s is the specific volume, p the pressure, u, v, w, three velocity components, and f the Coriolis parameter.

The basic state is a geostrophic flow and hydrostatic equilibrium, defined by

$$f\bar{u} = -\bar{s}\frac{\partial \bar{p}}{\partial y}\,,\quad g = -\bar{s}\frac{\partial \bar{p}}{\partial z}\,, \tag{3}$$

where the barred quantities denote those of the basic state. And we assume that the specific volume s, in general, depends on time, which means that the air density varies with time according to the distribution of heat sources and sinks. Then it may be assumed as follows:

$$\bar{s} = s_0 e^{l(z)t}, \tag{4}$$

where l is the factor corresponding to the non-adiabatic motion, assuming $l(z)\ t$ to be small, and s_0 represents the quantity corresponding to the adiabatic motion, expressed by means of the following equation:

$$\frac{d}{dt}\left(\frac{1}{s_0}\right) = \varepsilon \frac{dp}{dt}, \tag{5}$$

where $\varepsilon^{1/2}$ is the inverse of the Laplacian velocity of sound.

We assume that the perturbation, represented by u', v', w', p' and s' is superposed on this basic flow, then (2) and (5) become as follows:

$$\left\{\begin{aligned}
& L(u') + v'\bar{u}_y + w'\bar{u}_z + \bar{s}\frac{\partial p'}{\partial x} - fv' = 0 \\
& L(v') + fu' - f\bar{u}\frac{s'}{\bar{s}} + \bar{s}p_y' = 0 \\
& \bar{s}^2 \frac{\partial p'}{\partial z} - s'g = 0 \\
& \frac{1}{\bar{s}} L(s') - \frac{2s'}{\bar{s}^2}\frac{\partial \bar{s}}{\partial t} + \sigma_z v' + \sigma_z w' - \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}\right) = 0 \\
& \frac{1}{\bar{s}} L(s') + \sigma_y v' + \sigma_z w' + \bar{s}\,\varepsilon\,\{L(p') + v' p_y + w' p_z\} - \frac{s'}{\bar{s}^2}\frac{\partial \bar{s}}{\partial t} = 0,
\end{aligned}\right. \tag{6}$$

where the symbol L denotes the operator $\partial/\partial t + \bar{u}\partial/\partial x$, suffixes x, y and z denote partial differentiation with respect x, y and z respectively, and σ is defined by

$$\sigma \equiv \log s_0 \tag{7}$$

To simplify the discussion we shall assume that the wave disturbance has an infinite lateral extent, and that the geostrophic relation for the meridional velocity v' holds, (see CHARNEY 1947, p. 145), that is,

$$fv' = \bar{s}\,p_x'. \tag{8}$$

Also we assume that each perturbation quantity is expressed as a function of the harmonic factor, $\exp\left[i\,k\,(x - c\,t)\right]$, namely.

$$u', v', w', s', p' = U, V, W, S, P \exp\left[ik\,(x - ct)\right]. \tag{9}$$

Finally we get, (see KUO 1949),

$$-W = iksr\,(\bar{u} - c)\left[P_z + \sigma_z P\right] + iks\left(\frac{c}{g} - r\bar{u}_z\right)P - rl\,(z)\,s\,P_z, \tag{10}$$

and

$$\begin{aligned}
& rZ\,(\bar{u} - c - m)\,P_{zz} + rZ\left\{(\bar{u} - c)\,\sigma_z - m_z - \frac{m}{gr}\right\}P_z + rZ\left[\sigma_z \bar{u}_z - \bar{u}_{zz} + \sigma_{zz}\,(\bar{u} - c)\right]P \\
& + r_z Z\,\{(\bar{u} - c - m)\,P_z + (\bar{u} - c)\,\sigma_z P - u_z P\} + \left[\beta - (\bar{u} - c)\,k^2\right]P = 0.
\end{aligned} \tag{11}$$

where

$$m = \frac{l}{ik}, \quad \beta = \frac{\partial f}{\partial y} - \bar{u}_{yy}, \quad f\,(f - \bar{u}_y) = Z, \quad \frac{1}{r} = g\sigma_z - \varepsilon g^2. \tag{12}$$

In many cases the term $i\,k\,\mathfrak{s}\,cP/g$ in (10) becomes of lesser importance than the other terms, therefore it may be neglected as a rough approximation. The equation (10) then becomes to a simple formula,

$$\mathfrak{W} \equiv -\frac{W}{iks} = r\left[(\bar{u}-c-m)\,P_z + \{(\bar{u}-c)\,\sigma_z - u_z\}\,P\right]. \tag{13}$$

With substitution of this relation in (11), a differential equation for $\mathfrak{W}$ can be get :

$$\frac{d^2\mathfrak{W}}{dZ^2} + \left\{\sigma_z - \frac{u_z}{\bar{u}-c-\beta/k^2} - \frac{u_z - m\,\sigma_z}{\bar{u}-c-m}\right\}\frac{d\,\mathfrak{W}}{dz} + \frac{1}{rZ}\,\frac{\beta-(\bar{u}-c)k^2}{\bar{u}-c-m}\,\mathfrak{W} = 0\,. \tag{14}$$

A similar equation has been discussed by KUO (1952), who has assumed that r is constant and m is zero.

4 Numerical Method

The differential equation (14) can be integrated by the numerical method. With the substitution of,

$$1-\zeta = e^{-\sigma}, \quad \text{where } \sigma_{(z=0)} = 0\,. \tag{15}$$

we obtain,

$$(1-\zeta)^2\,\frac{d^2\mathfrak{W}}{d\,\zeta^2} + \left(\frac{\sigma_{zz}}{\sigma_z^{\,2}} - \frac{(1-\zeta)\,\bar{u}_\zeta}{\bar{u}-c-\beta/k^2} - \frac{(1-\zeta)\,\bar{u}_\zeta - m}{(\bar{u}-c-m)}\right)(1-\zeta)\,\frac{d\,\mathfrak{W}}{d\,\zeta}$$
$$-\frac{k^2}{rZ\sigma_z^{\,2}}\,\frac{\bar{u}-c-\beta/k^2}{\bar{u}-c-m}\,\mathfrak{W} = 0\,. \tag{16}$$

To carry out numerical computations, the values of σ_z, $1/r$ and s are needed. If we consider the atmospheric state whose surface temperature is 5°C and the lapse rate in the troposphere, 6°C/km, then the values of the necessary quantities become as shown in table 1. We will, however, assume the following two layers model, for the

Table 1 Numerical values for the atmosphere whose surface temperature is 5°C and the lapse rate in the troposphere is 6°C/km.

Altitude (km)	σ_z (m^{-1})	$1/r$ (sec^{-2})	σ	ζ
0	1.012×10^{-4}	1.382×10^{-4}	0	0
4	1.107	1.513	0.4242	0.3458
8	1.223	1.672	0.8898	0.5893
11 (troposphere)	1.314	1.798	1.270	0.7192
11 (stratosphere)	1.636	5.00	〃	〃
16	1.628	4.98	2.076	0.8746
20	1.620	4.96	2.727	0.9344

sake of simplicity : $\sigma_z = 1.142\times10^{-4}\ \mathrm{m}^{-1}$, $1/r = 1.562\times10^{-4}\ \mathrm{sec}^{-2}$ in the troposphere, $\sigma_z = 1.630\times10^{-4}$, $1/r = 5.00\times10^{-4}$ in the stratosphere and the tropopause being at $\zeta_T = 0.723$. These values are means of the respective values in table 1 both in the troposphere and the stratosphere.

By assuming no friction at the surface, we get the surface condition :

$$\mathfrak{W}=0\,,\quad \text{at}\quad \zeta=0\,. \tag{17}$$

The top boundary condition may be get of the consideration that no disturbance reaches there. That is to say :

$$\mathfrak{W}\to 0\,,\quad \frac{d\,\mathfrak{W}}{dz}\to 0\,,\quad \frac{d^2\,\mathfrak{W}}{dz^2}\to 0\,,\ \cdots\cdots,\quad \text{at}\ z\to\infty\,,$$

or

$$\mathfrak{W}\to 0\,,\quad \frac{d\,\mathfrak{W}}{d\,\zeta}=\text{finite}\,,\ \cdots\cdots,\quad \text{at}\ \zeta=1\,, \tag{18}$$

because

$$\lim_{z\to\infty}\frac{d\,\mathfrak{W}}{dz}=\lim_{\zeta\to 1}\sigma_z(1-\zeta)\frac{d\,\mathfrak{W}}{d\,\zeta}\,,\cdots\cdots.$$

The approximate solution may be expressed by the following finite power series of ζ :

$$\mathfrak{W}=a_0+a_1\zeta+\ \cdots\cdots\ +a_n\zeta^n\,. \tag{20}$$

$$\begin{cases} a_0=0 \\ a_1+a_2+\ \cdots\cdots\ +a_n=0 \end{cases} \tag{21}$$

When the boundary conditions are applied to (20), we get :

The approximate solution (20) does not satisfy the differential equation (16) and the left hand side of (16) has a value $\delta(\zeta)$. To determine coefficients of the finite series, we assume the relations :

$$\int_0^1\delta(\zeta)\,d\,\zeta=0\,,\cdots\cdots,\quad \int_0^1\delta(\zeta)\,\zeta^{n-1}\,d\,\zeta=0\,. \tag{22}$$

These relations are represented by linear equations of coefficients $a_1,\ \ldots,\ a_n$, provided $\bar{u}$ and m are functions of ζ, namely,

$$\begin{cases} b_{11}(c)\,a_1+b_{12}(c)\,a_2+\ \cdots\cdots\ +b_{1n}(c)\,a_n=0 \\ b_{21}(c)\,a_1+\ \cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots \end{cases} \tag{23}$$

In order that these homogeneous linear equations have non-trivial solutions, the following equation must be satisfied,

$$\begin{vmatrix} b_{11} & b_{12}\cdots\cdots & b_{1n} \\ b_{21} & \cdots\cdots\cdots & \\ \cdots\cdots & & \\ b_{n1} & \cdots\cdots\cdots & \end{vmatrix}=0\,. \tag{24}$$

This equation (24) is the equation of the eigen value problem. When c is real, the amplitude of the disturbance will not vary with time, and when c is complex the amplitude will vary with time.

We assume zonal wind to be :

$$\bar{u} = \bar{u}_0 (\zeta - \zeta^7), \quad \bar{u}_0 = 60\,\mathrm{m/sec}. \tag{25}$$

The zonal wind is maximum at $\zeta = 0.723$, at the tropopause.

On the other hand the rate of change of temperature is given by

$$\frac{\bar{s}}{c_p} \frac{dq}{dt} = T\bar{s}\,Q_2 .$$

Then we obtain :

$$\frac{d\bar{s}}{dt} = \frac{\bar{s}}{c_p} \frac{1}{T} \frac{dq}{dt} \;(= s\,Q_2) . \tag{26}$$

As was already discussed in section 2, the vertical distribution of Q_2 must be such as to agree with the estimated distribution when superposed on Q_1. Considering this circumstance and the easiness of computation, we will assume that

$$\bar{s}\,\frac{dq}{dt} = A \sin (\pi \zeta / \zeta_T), \quad \zeta_T \geqq \zeta \geqq 0 \tag{27}$$

where A is a constant, the value of A/ζ being assumed to be 0.7°C/day.

Then we have,

$$l = \frac{A}{c_p T} \sin (\pi \zeta / \zeta_T) . \tag{28}$$

The total change of heat of the air column is zero, because it is given by,

$$\int_0^\infty \frac{dq}{dt}\,dz = \int_0^{\zeta_T} \frac{A}{\bar{s}} \frac{\bar{s}}{\sigma_z \bar{s}_0} \sin (\pi \zeta / \zeta_T) = 0 .$$

5 The Results of Calculation

(1) $k = 1.48 \times 10^{-6}\,\mathrm{m}^{-1}$, (wave length = 4250 km).

This case nearly corresponds to the most unstable case and according to Kuo (1952) the wave doubles its amplitude within 24 hr.

At first we will consider the case in which $m=0$, which means that there are no heat sources and sinks. The values of the wave velocity, c. were obtained respectively for the first ($n=2$), second ($n=3$) and third ($n=4$) approximations.

$n = 2$		$c =$	$25.45 \pm 1.88i,$	$\mathrm{m\,sec^{-1}}$
$n = 3$		$c =$	$25.738 \pm 6.798i,$	"
	or	$=$	6.034 ,	"
	or	$=$	20.512 ,	"
$n = 4$		$c =$	$25.262 \pm 6.382i,$	"
	or	$=$	6.603 ,	"
	or	$=$	30.193 ,	"
	or	$=$	$15.639 \pm 2.753i.$	"

It is noticed that the number of the solution increases with increase of the degree of approximation, so that there are some doubt whether or not all the solutions are the eigen values. In this context, we consider that a solution is the eigen value if (16) is satisfied at several values of ζ between 0 and 1. According to this test we are able to show that $c=6.603$ m sec^{-1} is not the eigen value. As the consequence of the test the following three values remain to be the eigen value:

$$c = 30.193 \quad \text{m sec}^{-1}. \quad \text{stable wave}$$

$$c = 25.263 \pm 6.382i \quad '' \quad \text{unstable wave.}$$

The former stable wave has a velocity of the planetary wave having the wind velocity at the tropopause. The latter amplified and damped waves correspond to usual cyclonic waves with an amplifying factor of 0.82 per day. By (23) and (20) the vertical momentum of each wave are obtained as shown in fig. 2. It is seen that the

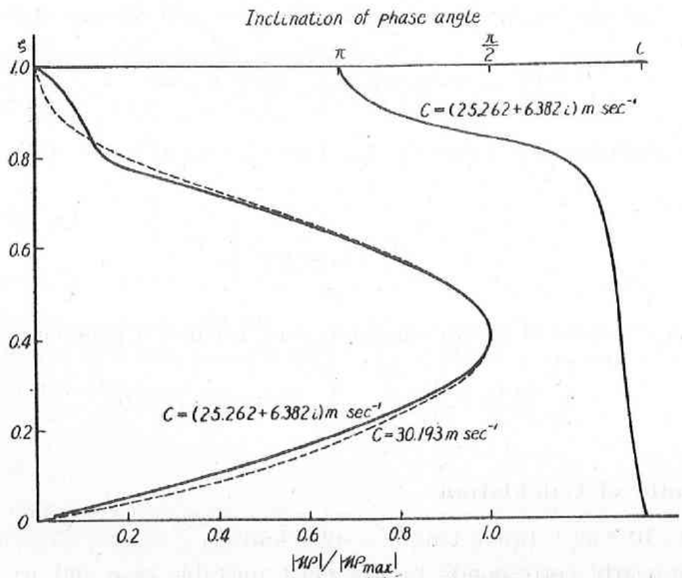

Fig. 2. The vertical momentum for $k=1.48\times10^{-6}$ m^{-1} and $m \neq 0$.

influence of the waves reaches the height of the tropopause. This fact reveals us that it is difficult to assume the tropoause to be a rigid boundary although such an assumption is often employed.

For $m=0$ and $n=3$, we have the following values:

$$c = 25.738 \pm 6.793i \quad \text{m sec}^{-1}$$

$$\text{or} \quad = 20.518 - 0.017i \quad ''$$

$$\text{or} \quad = 6.033 - 0.008i. \quad ''$$

The last solution is shown to be not an eigen function. Of the remaining three solutions, the first two are nearly same as those for $m=0$ and the third one has but a negligible damping factor. From the result we may infer that the influence of heat sources and sinks on the waves is so small that we can safely neglect the presence of heat sources and sinks.

(2) $k=8.00\times10^{-7}$ m^{-1}, (wave length $=7850$ km).

This case corresponds to that of the stable long wave.

For $m = 0$, we obtain :

$n = 2$		$c =$	2.249	m sec^{-1}
	or	$=$	27.202	〃
$n = 3$		$c =$	9.604	〃
	or	$=$	16.924	〃
	or	$=$	$16.185 \pm 8.067i$	〃

The second approximation ($n = 3$) differ remarkably from those of the first ($n = 2$). Examining these by the same way as before, we find that $c = 16.185 + 8.067i$ m sec^{-1} are not eigen values, so that two stable waves remain. The vertical momentum corresponding to these waves are shown in fig. 3. It is plausible to consider that $c = -9.603$ m sec^{-1} is the velocity of an usual planetary wave, and $c = 16.924$ m sec^{-1} is that of the planetary wave having the wave velocity at the tropopause.

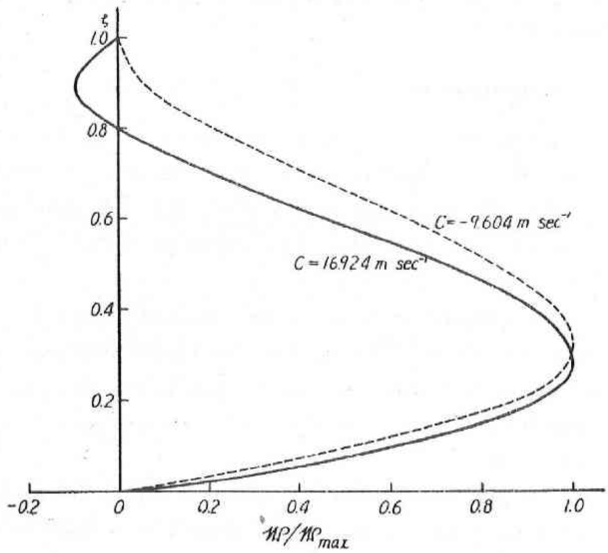

Fig. 3. The vertical momentum for $k = 8.00 \times 10^{-7}$ m^{-1} and $m \neq 0$.

For $m = 0$, $n = 3$, we have :

	$c =$	$-9.604 + 0.007i$	m sec^{-1}
or	$=$	$16.924 + 0.073i$	〃
or	$=$	$16.189 + 8.067i$	〃
or	$=$	$16.181 - 8.126i.$	〃

The last two solutions are not eigen values by the same examination as before. The first two remain as eigen values which correspond to slightly ampilfied wave with the amplifying factor of 0.0005 per day and 0.0051 per day respectively. It is remaked that both in the case (1) with $k = 1.48 \times 10^{-6}$ m^{-1} and in the present case (2) $k = 8.00 \times 10^{-7}$ m^{-1} the presence of waves whose velocity corresponds to the wind velocity of the tropopuase has been resulted in our computations, but the presence of such waves is still in debate.

(3) The single layer model

We will consider the single layer model $\sigma_z = 1.142 \times 10^{-4}$ m^{-1} and $1/r = 1.562 \times 10^{4}$ sec^{-2}, in order to find the influence of the stratosphere. For $m = 0$, $n = 3$, we have :

$k = 1.48 \times 10^{-6}\ \mathrm{m}^{-1}$. $\quad c = 24.933 \pm 4.871i \quad \mathrm{m\ sec}^{-1}$.

or $\quad = 7.124 \quad$ 〃

or $\quad = 21.475 \quad$ 〃

$k = 8.00 \times 10^{-7}\ \mathrm{m}^{-1}$. $\quad c = -9.264 \quad$ 〃

or $\quad = 18.362 \quad$ 〃

or $\quad = 16.670 \pm 6.911i \quad$ 〃

From these results we find that for long waves the influence is so small and can be neglected in crude discussions and that for cyclonic wave the influence is rather greater.

6 Conclusions

Comparing the computations for $m=0$ and $m \neq 0$ it appears that the influence of vertically distributed non-adiabatic heating and cooling, Q_2, on the zonal flow is to amplify the long planetary wave. But this influence is so small as to be appreciable in short time intervals. The influence of large scale heat sources and sinks, Q_1, considered by SMAGORINSKY is also small and of the same magnitude as the influence of the orographic influence. He concluded that his quantitative study does not conclusively support whether the orographic influence dominates or the thermal influence does. At any rate it will be concluded that the presence of Q_2 playes a role of amplifying the long planetary wave in opposition to the frictional decay effect of small scale eddies.

On the other hand the effect of the stratosphere is important. The single layer approximations with the repspective simplifications of the finite atmosphere bounded by two ridid surfaces and of the infinite atmosphere, are not satisfactory for especially unstable wave.

Acknowledgements — The writer wishes to thank to Professor G. YAMAMOTO for his advices and many constructive suggestions.

Refferences

CHARNEY, J. G. 1947: The dynamics of long waves in a baroclinic westerly current. *J. Met.* **4**, 135–162.

HOUGHTON, H. G. 1954: On the annual heat balance of the northern hemisphere. *J.Met.* **11**, 1–9.

KUO, H. L. 1949: Dynamic instability of two-dimensional nondivergent flow in a barotropic atmosphere. *J. Met*, **6**, 105–122.

——— 1952: Three-dimensional disturbances in a baroclinic zonal current. *J. Met.* **9**, 260–278.

LONDON, J. 1952: The distribution of radiational temperature change in the northern hemisphere during March. *J. Met.* **9**, 145–151.

——— 1953: The distribution of infra-red cooling in the atmosphere for winter and summer seasons. *Proc. Toront Met. Confer.* 60–70.

RAETHJEN, P. 1950: *Warmeshaushalt der Atmosphäre.* Cited by GOODY, R. M. & ROBINSON, G. D., *Q. J. Roy. Met. Soc.* **77**, (1951), 151–187.

SMAGORINSKY, J. 1953: The dynamical influence of large-scale heat sources and sinks on the quasi-stationary mean motions of the atmosphere. *Q. J. Roy. Met. Soc.* **79**, 342–366.